# BASICS OF PROBABILITY

**1** Fill in he following table.

| putting stake s on | paid back |
|---|---|
| MANQUE/PASSE | **2s** |
| PAIR/IMPAIR | **2s** |
| ROUGE/NOIR | **2s** |
| $12^D$ / $12^M$ / $12^P$ | **3s** |
| 1st, 2nd, 3rd column | **3s** |
| single number | **36s** |
| square | **9s** |

| | |
|---|---|
| MANQUE | the numbers from 1 to 18. |
| PASSE | the numbers from 19 to 36. |
| PAIR | the even numbers. |
| IMPAIR | the odd numbers. |
| ROUGE | the red numbers. |
| NOIR | the black numbers. |
| $12^D$ | the numbers from 1 to 12. |
| $12^M$ | the numbers from 13 to 24. |
| $12^P$ | the numbers from 25 to 36. |
| square | four numbers that join the same vertex. |

**2** Calculate the probability of the events A, B, C and D from above:

a) $A = \text{red numbers} = \{1, 3, 5, 7, 9, 12, 14, \dots, 36\}$.

b) $B = \text{2nd column} = \{2, 5, 8, \dots, 35\}$.

c) $C = 12^P = \{25, 26, \dots, 36\}$.

d) $D = \text{number } 14 = \{14\}$.

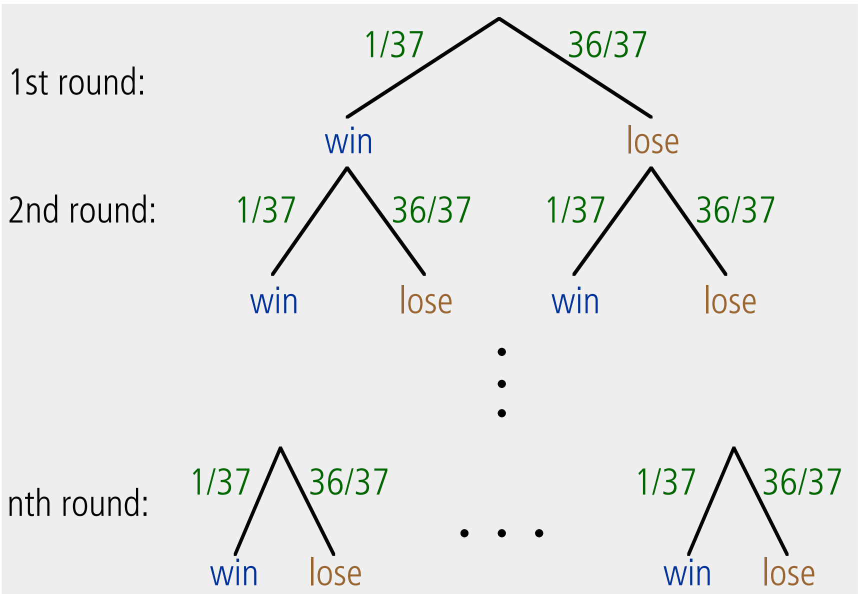
a) $|A| = 18 \quad \Rightarrow \quad p(A) = \frac{|A|}{|\Omega|} = \mathbf{\frac{18}{37}}$

b) $|B| = 12 \quad \Rightarrow \quad p(B) = \frac{|B|}{|\Omega|} = \mathbf{\frac{12}{37}}$

c) $|C| = 12 \quad \Rightarrow \quad p(C) = \frac{|C|}{|\Omega|} = \mathbf{\frac{12}{37}}$

d) $|D| = 1 \quad \Rightarrow \quad p(D) = \frac{|D|}{|\Omega|} = \mathbf{\frac{1}{37}}$

**3** At roulette, how many times do you have to put money on the same number to win at least once with a probability of at least 95 %?




The probabilities of most events formed with the words "at least" or "not more than" are difficult. Often it is much easier to calculate the probability of the complement.

A = "winning at least once" $\Rightarrow$ $\left|\bar{A}\right|$ = "losing n times in a row"

$$p\left(\bar{A}\right)=\left(\frac{36}{37}\right)^n \quad \Rightarrow \quad p\left(A\right)=1-p\left(\bar{A}\right)=1-\left(\frac{36}{37}\right)^n \geq 95\ \%$$

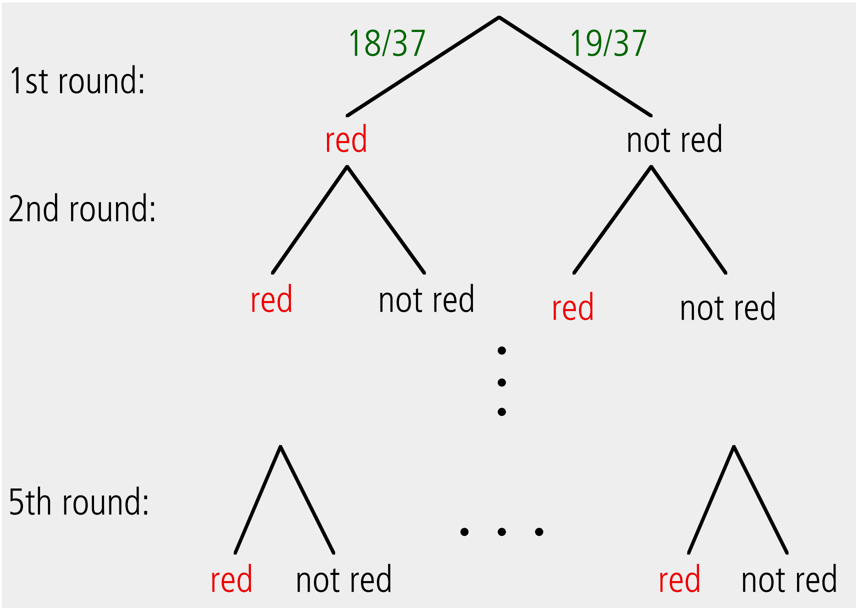
$$\Leftrightarrow \quad 1-\left(\frac{36}{37}\right)^n \geq 0.95$$

Solve $\Rightarrow$ $n \geq$ **110**

**4** At roulette, what is the probability that

a) five times in a row a red number shows up?

b) in the 5th round red shows up under the condition that in the four previous rounds there were four red numbers in a row?



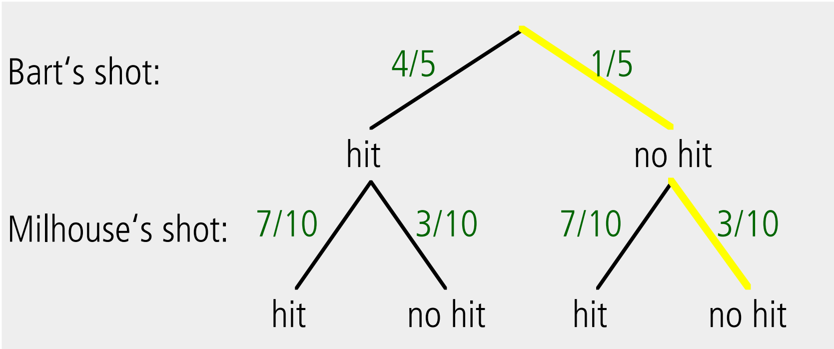

a) $p(\text{"five times red in a row"}) = \left(\frac{18}{37}\right)^5 = \mathbf{0.03} = \mathbf{3\ \%}$

b) $p(\text{"red when there were four red in a row"}) = \frac{18}{37} = \mathbf{0.49} = \mathbf{49\ \%}$

The ball has no memory and does not know what colour it was in the previous rounds.

**5**

Bart and Milhouse try out their catapult and shoot one pebble each at Principal Skinner's office window. Bart usually hits the target four times out of five, Milhouse is less skilful and hits it only seven times out of ten. What is the probability that the window is hit at least once?




A = "hitting at least once" $\Rightarrow$ $\left|\overline{A}\right|$ = "not hitting"

$$p\left(\overline{A}\right)=\frac{1}{5}\cdot\frac{3}{10}=\frac{3}{50} \quad\Rightarrow\quad p\left(A\right)=1-p\left(\overline{A}\right)=1-\frac{3}{50}=\frac{47}{50}=\mathbf{0.94}=\mathbf{94\ \%}$$

**6** Mr Bond claims to be able to distinguish a stirred from a shaken Martini by tasting only. His friends do not believe him and want to put him to the test. They set up a row of five identical glasses and fill them at random with either a stirred or a shaken Martini. Then they ask Mr Bond to taste blindfolded and pick out the stirred ones. They will acknowledge his ability if he gets at least four glasses right.

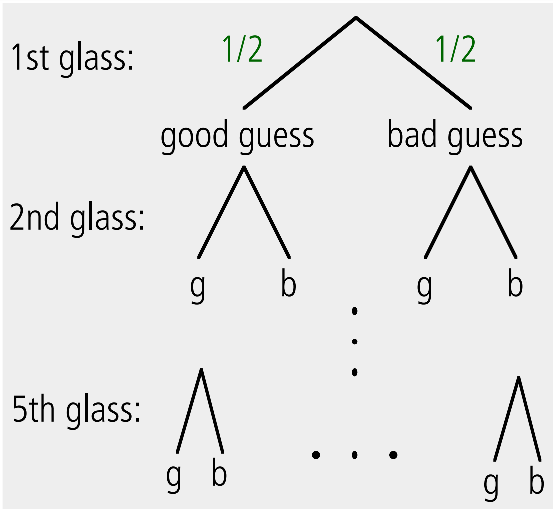
a) What is the probability for Mr Bond to pass the test by just guessing?

b) What is the probability for Mr Bond to fail the test even if he can tell the glasses apart with a probability 90 %?

a) p("at least four right")
= p("five right") + p("four right")

$$=\left(\frac{1}{2}\right)^5+5\left(\frac{1}{2}\right)^5$$

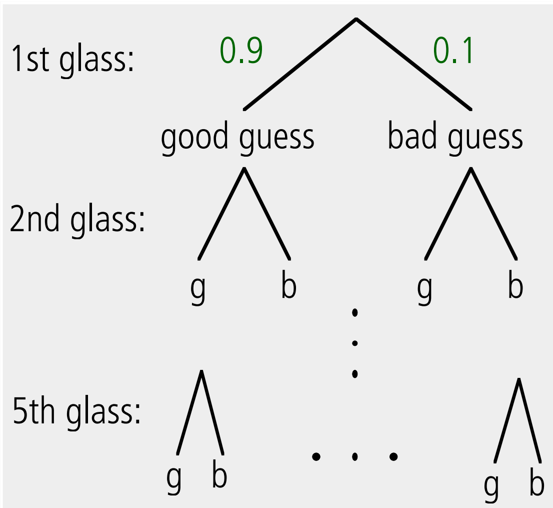
$$=\mathbf{\frac{6}{32}}=\mathbf{18.75\ \%}$$




b) p("fewer than four right")
= 1 - p("five right") - p("four right")

$$=1-0.9^5-5\cdot 0.9^4\cdot 0.1$$

$= 0.081 = 8.1$ **%**



**7**

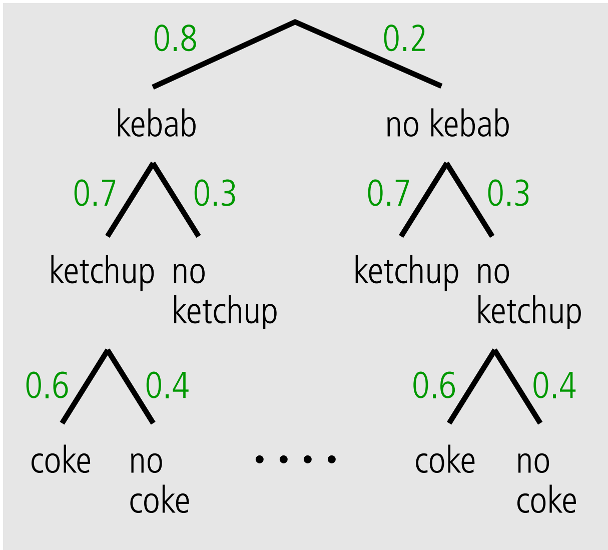
The owner of a fast-food place knows from experience that 80 % of his customers order kebab, 70 % want some extra ketchup and 60 % have a coke to wash it down. To reduce costs he decides to serve everybody kebab with ketchup and a coke without even asking.

a) What are the chances that a customer gets exactly what he wants?

b) What are the chances that the customer gets at least two things he wants?

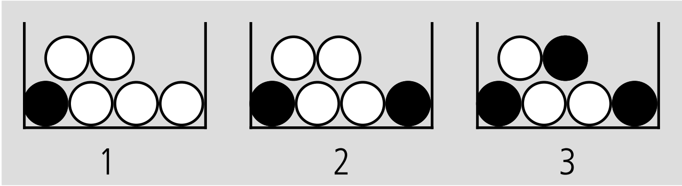
a) p(everything is all right)
$= 0.8 \cdot 0.7 \cdot 0.6$
$= 0.336 = 33.6\ \%$

b) p(at least 2 things are all right)
= p(everything is all right)
+ p(exactly 2 things are all right)
$= 0.8 \cdot 0.7 \cdot 0.6$
$+ 0.8 \cdot 0.7 \cdot 0.4$
$+ 0.8 \cdot 0.3 \cdot 0.6$
$+ 0.2 \cdot 0.7 \cdot 0.6 = 0.788 \approx 78.8\ \%$



**8** The caliph of Baghdad had the habit of having thieves incarcerated and the right hand chopped off. But beforehand they got a chance to save the limb. They could choose blindfolded one of the three urns and draw a ball. If the ball was white they where freed immediately.

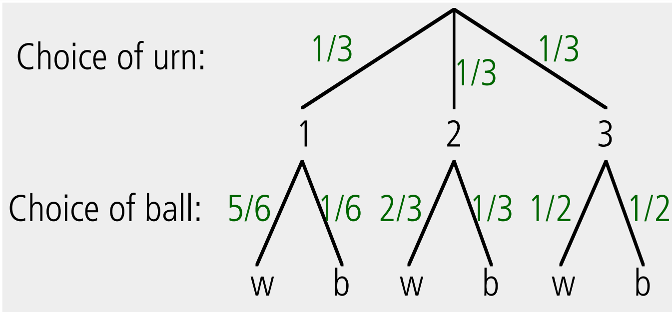
a) What is the probability to draw a white ball?




One day a clever thief asked whether he was allowed to move some balls - with eyes open - from one urn to the other. The caliph agreed assuming that this would not increase the thief's chance to escape unharmed.

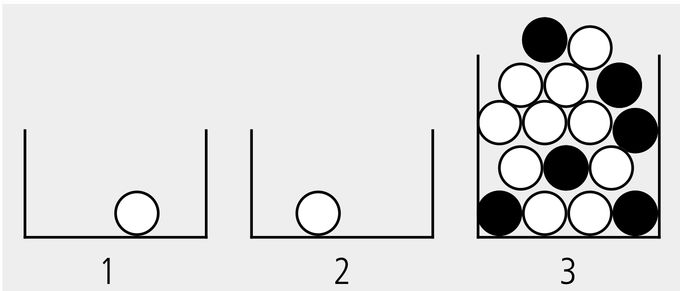
b) Was the caliph right?

a) The experiment is divided into two stages:

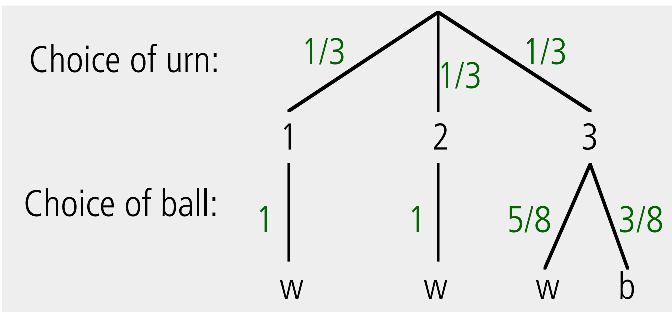



$$p(\text{"white ball"}) = \frac{1}{3}\cdot\frac{5}{6} + \frac{1}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{1}{2} = \frac{5+4+3}{18} = \frac{12}{18} = \mathbf{\frac{2}{3}}$$

b) The clever thief moved the balls like this:




Then the probability tree changes into:




$$p(\text{"white ball"}) = \frac{1}{3}\cdot 1 + \frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{5}{8} = \frac{8+8+5}{24} = \mathbf{\frac{7}{8}}$$

The caliph was **not** right.


**9** The "one-armed bandit" is a special kind of gambling machine in casinos. Basically there are three spinning drums with - in our case - ten different pictures on each of them. By pulling the lever (= arm) down the drums are set in motion and start spinning. They come to a random stop independently after a certain time and show a picture each.

a) You hit the jackpot if the three pictures are identical. What are the chances?

b) What are the chances that exactly two pictures are identical?

a) A = all three pictures are identical

$|A|$ = number of possibilities to arrange linearly 1 picture out of 10

$$= {}^{10}P_1 = \frac{10!}{(10-1)!} = 10$$

$|\Omega|$ = number of possibilities to arrange linearly 3 pictures out of 10 (repetitions possible)

$$= {}^{10}\tilde{P}_3 = 10^3 = 1{,}000$$

$$p(A) = \frac{|A|}{|\Omega|} = \frac{10}{1{,}000} = 0.01 = 1\,\%$$

b) B = exactly two pictures are identical

$|B|$ = number of possibilities to arrange linearly 2 pictures out of 10 (the first number is the one that appears twice) multiplied by the number of possibilities to arrange these three pictures linearly

$$= {}^{10}P_2 \cdot {}^{3}P_2 = \frac{10!}{(10-2)!} \cdot \frac{3!}{2!} = 90 \cdot 3 = 270$$

$$p(A) = \frac{|A|}{|\Omega|} = \frac{270}{1{,}000} = 0.27 = 27\,\%$$

**10** In a bag there are 26 wooden cubes with a different letter of the alphabet on each one. Now the bag is shaken to shuffle the cubes thoroughly. Then, one by one, 4 cubes are randomly taken out.


a) What are the chances that in the order they are taken out they form the word "MARS"?

b) What are the chances that the word "MARS"can be formed by them?

a) A = the letters form the word "MARS" in the order they are taken

$$|A| = 1$$

$$|\Omega| = {}^{26}P_4 = \frac{26!}{(26-4)!} = 358{,}800$$

$$p(A) = \frac{|A|}{|\Omega|} = \frac{1}{358{,}800} \approx 0.0000028 = 0.00028\ \%$$

b) B = the word "MARS" can be formed

$$|B| = {}^{4}P_4 = 4! = 24$$

$$|\Omega| = 358{,}800$$

$$p(B) = \frac{|B|}{|\Omega|} = \frac{24}{358{,}800} \approx 0.000067 = 0.0067\ \%$$

or

$$|B| = 1$$

$$|\Omega| = {}^{12}C_4 = \binom{26}{4} = 14{,}950$$

$$p(B) = \frac{|B|}{|\Omega|} = \frac{1}{14{,}950} \approx 0.000067 = 0.0067\ \%$$

**11** "Texas Hold'm" is a form of poker in which every player is given randomly five cards out of 52. In addition two more cards which belong to all players are openly placed on the table.

a) What are the chances for "All Fours" (four cards of the same rank, e.g. four nines)?

b) What are the chances for a "Full House" (three cards of the same rank and two cards of the same rank, e.g. three aces and two eights)?

It does not matter whether the cards are on the hand or on the table. The order has no influence either.

a) A = All Fours

$$|A| = 13 \cdot {}^{48}C_3 = 13 \cdot 17{,}296 = 224{,}848$$

$$|\Omega| = {}^{52}C_7 = 133{,}784{,}560$$

$$p(A) = \frac{|A|}{|\Omega|} = \frac{224{,}848}{133{,}784{,}560} \approx 0.002 = 0.20\ \%$$

b) B = Full House

$$|B| = 13 \cdot {}^{4}C_3 \cdot 12 \cdot {}^{4}C_2 \cdot {}^{44}C_2 = 3{,}541{,}824$$

$$p(B) = \frac{|B|}{|\Omega|} = \frac{3{,}541{,}824}{133{,}784{,}560} \approx 0.0285 = 2.85\ \%$$